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Aeroelastic Design Optimization of a Two-Spar Flexible Wind-Tunnel Model

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Introduction

AS part of an effort considering aerodynamic concepts for future aircraft,[1] the potential of using microflaps for aeroelastic control will be demonstrated in wind-tunnel testing. A cantilever wing-type wind-tunnel model with several design requirements is considered for this purpose. The low-speed wind-tunnel facility in which the testing will be performed imposes constraints on the flutter speed and structural strength, whereas the bandwidth of the electrically actuated flaps imposes an upper bound on the flutter frequency. Even when the influence from the flaps are left out of the problem, which is the case in this study, these constraints result in a fairly difficult aeroelastic design problem.

Aeroelastic wind-tunnel models are usually designed to represent the aeroelastic behavior of full-scale structures. The behavior of the model is then related to that of the full-scale structure by certain scaling rules.[2] As discussed by French and Eastep,[3] the scaling procedure is simple, although a difficult and highly constrained model design problem often results. This is not a concern in the present case, where the requirements are specified for the wind-tunnel model itself. In this Note, the feasibility of one possible concept for the model structural design is investigated by solving an optimization problem in which the structural weight is minimized subject to the aeroelastic constraints.

Two-Spar Design

The problem stated is to design a wing-type wind-tunnel model with a flutter speed $u_F = \hat{u}_F = 46$ m/s (150 ft/s), divergence speed $u_D \geq \hat{u}_D = 61$ m/s (200 ft/s), flutter frequency $f_F \leq \hat{f}_F = 3$ Hz, and a safety factor $n_S = 5$ for the static structural stresses at $u = \hat{u}_F$ (the flutter speed) due to an initial angle of attack $\theta_S = 10$ deg.

One of the concepts for the model structural design that was studied is shown in Fig. 1. A clamped wing configuration is considered, for which the elastic behavior is assumed to be dominated by the bending deformation of two composite spars. If the torsional rigidity is given by differential bending only, a configuration composed of two identical spars would have coalescing bending and torsion eigenfrequencies. This implies that a very low-frequency, first bending/first torsion type of flutter could result for a wing with this structural design.

An unswept rectangular planform with semispan 1.8 m (6 ft) and aspect ratio 7 is considered. Each composite spar is assumed to have a circular cross section with piecewise constant diameter, such that the wing is structurally divided into four spanwise partitions of equal length. Furthermore, the discrete values of the spar dimensions are assumed to vary in a linear fashion along the span. The spar configuration is, thus, determined by six design variables: the root and tip diameters of the front and rear spars (four design variables), and the chordwise locations x_f and x_r of the spars (two design variables).

The aerodynamic loads are transferred to the spars through a solid foam airfoil with embedded ribs. To minimize the influence on the torsional stiffness used in the model, the foam and ribs are assumed not to be attached to the spars. The ribs also serve as an attachment for a mass balancing composed of 16 concentrated masses, giving a total of 22 design variables.

Aeroelastic Analysis

A reasonably accurate structural analysis (for the purpose of this study) is obtained by modeling the spars and the foam airfoil as Euler beams. The result is a three-beam model having constant (but different) locations of the inertial and elastic axis in each wing partition. This is indicated for the elastic axis shown in between the spars in Fig. 1.

Equations of Motion

The linear equations of motion for the deflection $w(y, t)$ and twist angle $\theta(y, t)$ of the elastic axis in each wing partition may be written in the form

$$m\ddot{w} - ms\ddot{\theta} + EIw'''' = L \qquad (1)$$

$$J\ddot{\theta} - ms\ddot{w} - (GK)_a\theta'' + \epsilon\theta'''' = M \qquad (2)$$

where a dot denotes differentiation with respect to time t and a prime denotes differentiation with respect to y. The mass and mass moment of inertia (with respect to the elastic axis) per unit span are denoted by m and J, respectively. The amount of inertial bending/torsion

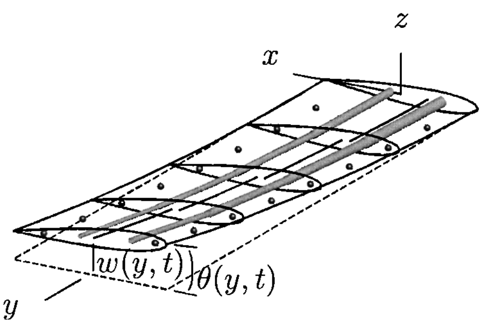

Fig. 1 Schematic layout of the two-spar design; note that only every other rib is shown.

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coupling is determined by the chordwise separation s (with sign) between the inertial axis and the elastic axis, EI is the total bending stiffness, and $(GK)_a$ the torsional rigidity of the foam airfoil. The differential bending portion of the torsional stiffness (to which all three beams contribute) is determined by the second moment of bending stiffness[2]

$$\epsilon = \sum_{j=1}^{3} l_j^2 (EI)_j \tag{3}$$

where $(EI)_j$ is the bending stiffness of beam j and l_j is the distance from the beam to the common elastic axis. The aerodynamic loads acting on the wing are represented by the lift $L(y, t)$ per unit span and pitching moment $M(y, t)$ per unit span with respect to the elastic axis. Finally, an assumption that the elastic axes of the adjacent partitions are chordwise rigidly connected to each other at their intersection results in a set of linear kinematic constraints that couples the corresponding equations of motion.

Numerical Analysis

To obtain a simple but still useful aeroelastic analysis, Theodorsen's unsteady lift and pitching moment for simple harmonic motion (see Ref. 2) are used for the spanwise loading. When the resulting equations of motion are discretized using beam finite elements with the nodal degrees of freedom $\{w \;\; w' \;\; \theta \;\; \theta'\}$ and transformed to the frequency domain, the familiar nonlinear eigenvalue problem

$$[\boldsymbol{M}p^2 + \boldsymbol{K} - q\boldsymbol{A}(k)]\boldsymbol{v} = \boldsymbol{0} \tag{4}$$

is obtained. In the eigenvalue problem Eq. (4), $\boldsymbol{M}$ is the consistent mass matrix, $\boldsymbol{K}$ the stiffness matrix, $\boldsymbol{A}(k)$ the matrix of aerodynamic forces, p the eigenvalue, and $\boldsymbol{v}$ the mode shape of the nodal degrees of freedom in the finite element model. The nonlinearity is due to the dependence on the reduced frequency of vibration $k = \omega b/u$, b denoting the wing semichord and u the airspeed, because the frequency of vibration ω is the imaginary part of the eigenvalue p. The scalar $q = \rho u^2/2$ is the dynamic pressure and ρ the air density.

A more computationally efficient flutter analysis was obtained by the use of a standard modal formulation[4] based on the first few free-vibration modeshapes. Furthermore, a uniform modal damping model was introduced to account approximately for structural damping.[5] When the nonlinear eigenvalue problem is solved using the p–k method,[6] the wing is considered stable if all eigenvalues have negative real parts. In particular, the divergence speed is obtained by solving for the smallest dynamic pressure, giving an eigenvalue $p = 0$, which is the smallest real solution to the eigenvalue problem

$$[\boldsymbol{K} - q\boldsymbol{A}(0)]\boldsymbol{v} = \boldsymbol{0} \tag{5}$$

The requirements on the structural strength are approximately accounted for by simply posing constraints on the beam stresses due to static aeroelastic deformation of the wing. Denoting the vector of design variables $\boldsymbol{x}$, the static aeroelastic deformation $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x})$ of the elastic axis due to a specified initial angle of attack θ_S is computed by solving the system of equations

$$[\boldsymbol{K} - q\boldsymbol{A}(0)]\boldsymbol{v} = q\boldsymbol{A}(0)\boldsymbol{v}_S \tag{6}$$

where $\boldsymbol{v}_S$ is a vector describing the initial (nodal) geometry of the wing. The nodal stresses $\boldsymbol{\sigma}(\boldsymbol{x})$ in the outer fiber of the spars can then be related to the deformation of the elastic axis through

$$\boldsymbol{\sigma} = \boldsymbol{S}\boldsymbol{v} \tag{7}$$

where $\boldsymbol{S}(\boldsymbol{x})$ is a stress matrix that depends on all spar-type design variables.

Design Optimization

One possible approach to solving the design problem is to pose an optimization problem where the structural weight is minimized subject to aeroelastic and stress constraints. The idea is that reducing mass and structural stiffness will drive the model to flutter at the specified critical speed, while at the same time being feasible with respect to the other design constraints. Note that the mass balancing has no influence on static stress, but does affect the flutter speed and frequency.

The described optimization problem may, in a somewhat simplified form, be posed as the nonlinear programming problem

$$\min_{\boldsymbol{x}} W(\boldsymbol{x}) \tag{8}$$

$$\mathrm{Re}\, p_F(\boldsymbol{x}, \hat{u}_F) \le 0 \tag{9}$$

$$\mathrm{Im}\, p_F(\boldsymbol{x}, \hat{u}_F) \le \hat{\omega}_F \tag{10}$$

$$u_D(\boldsymbol{x}) \ge \hat{u}_D \tag{11}$$

$$\sigma_j(\boldsymbol{x}, \theta_S, \hat{u}_F) \le \sigma_S/n_S \tag{12}$$

$$x_r - x_f \ge d \tag{13}$$

where $\boldsymbol{x}$ is the vector of design variables, $W(\boldsymbol{x})$ the model weight, p_F the flutter eigenvalue obtained from Eq. (4), $\hat{\omega}_F$ the maximum feasible flutter frequency, u_D the divergence speed obtained from Eq. (5), σ_j the spar stresses in the root of each wing partition ($j = 1, \ldots, 8$), and σ_S the failure stress of the spar composite material. Finally, for the differential bending principle to work well in practice, a minimum feasible distance d between the spars is enforced. This constraint also ensures that the two spars are not confused during the optimization.

In general, several flutter eigenvalues have to be considered in the optimization because it is not known in advance which eigenvalue is critical at optimum (if any). Furthermore, the stability constraints are to be enforced for subcritical airspeeds to ensure stability up to $\hat{u}_F$. Also note that the bound constraints on the design variables are not presented.

The optimization problem [Eqs. (8–13)] is solved using the method of moving asymptotes developed by Svanberg.[7] Semianalytical derivatives of the constraint functions are derived as described by Haftka and Adelman[8] and Ringertz.[9] The derivatives of the eigenvalue constraints exist provided that the eigenvalues are distinct.[10]

The numerical analysis was based on 32 finite elements and 8 free-vibration modes in the modal flutter analysis. The minimum feasible distance between the spars was set to $d = 2b/3$ and a 10% uniform modal damping was used. A significant amount of damping was expected due to the very high damping observed for the foam material.

Results

Initial optimization studies led to a design with a weak uniform rear spar and a more robust tapered front spar, with a first bending/first torsion type of flutter. To simplify manufacturing, the decision was made to use available dimensions of the composite spars to realize the tapering. Consequently, the rear spar diameter and the front spar tip and root diameters were fixed to 13 mm ($\frac{1}{2}$ in.), 19 mm ($\frac{3}{4}$ in.) and 38 mm ($\frac{3}{2}$ in.), respectively. The number of design variables was, thus, reduced to 18 for the final design optimization.

The objective of the final optimization study was to find a minimum weight mass balancing, using the concentrated masses and spar locations as design variables, subject to the constraints Eqs. (9–13). An upper bound of 1 kg for each concentrated mass (reasonable to fit into the model) was found sufficient to reduce the flutter frequency to the required level.

The resulting optimal design is shown in Fig 2. The spars are located slightly aft quarter and midchord, respectively, being separated by the minimum feasible distance $2b/3$. As can be observed in Fig. 2, only 3 of the 16 concentrated masses remain for the optimal

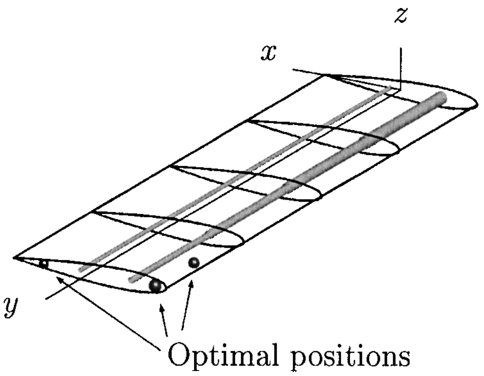

Fig. 2 Optimal design.

design. The maximum feasible mass, 1 kg, is located at the wing-tip leading-edge position, 0.70 kg at the inner leading-edge position, and 0.49 kg at the wing-tip trailing-edge position.

Both the flutter stability and frequency constraints in Eqs. (9) and (10) are active for the optimal design, meaning that both the target flutter speed and frequency are reached. At the flutter speed, a first torsion/first bending type of flutter occurs, whereas the other modes show a significantly higher damping for the considered speed range. The divergence speed for the optimal design is $u_D = 84$ m/s (274 ft/s). This is higher than the minimum feasible value $\hat{u}_D = 61$ m/s (200 ft/s), and the static stability constraint Eq. (11) is not active. However, the static stress at the root of the front spar is the maximum feasible, and the constraint in Eq. (12) representing this location is active. To summarize, a feasible design has been obtained.

Conclusions

The optimization formulation of minimizing structural weight was successfully applied to the present aeroelastic design problem, and a feasible design was obtained using numerical optimization. However, the design process was not without flaws. Whereas the low torsional stiffness of the two-spar design enabled a low flutter speed and frequency, the wing was also very prone to divergence. Taking wing sweep into account revealed that no significant improvement was to be expected for moderate sweep. Instead, the optimization resolved this obstacle by tapering the front spar only, which increased the divergence speed through bending/torsion coupling.

Another interesting feature of the optimal design is that the mass balancing tends to inertially decouple the two first modes of vibration of the unbalanced configuration. Without mass balancing, the first two modes of vibration are strongly influenced by both bending and torsion, mainly due to the coupling introduced by the tapered front spar. With mass balancing, the wing displays almost decoupled first bending and torsion modes.

Also note that the influence of the microflaps has not been taken into account in this study, and it is recommended that the analysis be extended to include a representative model of the actuators. The main conclusion is that the two-spar concept may be an option for the structural design of the wind-tunnel model. However, care must be taken with respect to the approximations made, and the model is fairly complex to realize. Other possible concepts, such as a plate design, may be more tractable. Of course, the present optimization-based approach to aeroelastic design may be useful in future studies as well.

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Speed-Sensitivity Analysis by a Genetic Multiobjective Optimization Technique

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Introduction

THE availability of effective tools that quickly provide aircraft overall characteristics sensitivity for different figures of merit is a very important factor during the early phase of the design process (that is, conceptual design phase). Typical figures of merit used are the following ones: gross weight, empty weight, fuel burned, and cruise speed. A multiobjective optimization technique can be used to understand how optimum configurations change as different objectives are selected. An example of such a parametric multiobjective approach can be found in Ref. 1. In this mentioned work a global figure of merit is defined as a weighted sum of selected objective functions. An effective gradient-based optimization technique[2,3] is used, and different design solutions are obtained by changing the weight value. This Note deals with the application of a genetic multiobjective optimization technique in the field of aircraft requirements analysis. This procedure takes advantage of the well-known genetic parallel-like searching method and allows us to obtain sensitivity curves by only one optimization run. Once a specific requirement has been selected (for example, range, speed, ceiling, takeoff distance, etc.), these curves provide a deeper understanding of the requirement effect on the aircraft configuration. In particular, cruise speed effect has been evaluated in this Note. Such a type of procedure can thus be proposed as a very useful and effective tool for tradeoff studies aimed at the final freeze of requirements. Classical

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